

The Chinese South-Seeking chariot: A simple mechanical device for visualizing curvature and parallel transport

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(Received 16 October 1991; accepted 19 February 1992)

An old mechanical device, the Chinese South-Seeking chariot, presumably designed to work on a flat plane, is shown to perform parallel transport on arbitrary surfaces. Its use affords experimental demonstration and even numerical checking (within a reasonable accuracy) of all the features of curvature and parallel transport of vectors in a two-dimensional surface.

I. INTRODUCTION

Curvature and parallel transport are two mathematically important concepts playing an increasing role in physics. In Einstein's theory of gravitation, curvature of space-time is related (through Einstein's equations) with the density of energy.¹ Other structures involving a kind of parallel transport have been recognized as relevant in electromagnetism, Yang-Mills, and gauge theories. There are also more elementary examples where parallel transport plays a relevant role—for instance Foucault's pendulum² or noninertial frames in Newtonian physics.³ The study of geometric phases, starting with Berry's paper,⁴ has shown a startling variety of physical situations where these phases are important;⁵ they can be also considered as direct manifestations of specific parallel transports in some bundles that can be used to describe the system under study.

The relevance of these concepts makes it advisable to introduce them to physics students as soon as possible. But even the idea in its basic form—parallel transport for vectors in surfaces—requires some moderately advanced mathematical tools. However, as Kugler pointed out in this Journal,³ while these concepts are usually taught in general relativity courses, they can be readily understood (without complete proofs) at an earlier stage, using simple experiments as demonstrations during lectures.

In this paper we describe how an old mechanical device, the Chinese South-Seeking Chariot (hereafter SSC) can be successfully used to explore, by doing some "experimental (physical) geometry," the basic ideas of Gaussian curvature of a surface and of parallel transport of a vector along a curve. The key observation is that (in the limit of "zero size"), the SSC performs Levi-Civita parallel transport for the direction of its pointer when moving along a curve on any surface. As far as I am aware, this use of the SSC as a "machine of parallel transport" has not been pointed out previously.

Curvature is a mathematical concept; nevertheless the exposition in this paper makes appeal to some kind of geometrical "experimentation." I wish to stress that here the geometry of surfaces is considered as a physical theory, which could and should be the object of experimentation and measurement just as any other branch of physics. In this context, experimentation with the SSC may help the students to develop very easily the correct intuitions. Previously known results in the geometry of surfaces can be easily checked; alternatively the SSC may be used as a demonstration apparatus, only to let the students grasp the main concepts before a more formal study.

Historically the starting point of the concept of curvature was the theory of surfaces in 3-D Euclidean space. But

it took a long time to come to the conclusion that parallel transport is the fundamental idea;⁶ as a matter of fact, the concept of parallel transport arose only very lately. Here the essential facts are:

(a) Geodesics in the surface can be characterized either as lines of (locally) least length between two points, or as lines whose tangent vector undergoes parallel transport along themselves. Parallel transport of any vector tangent to the surface along a geodesic can be obtained by requiring it to be a vector of constant modulus whose angle with the geodesic remains constant.

(b) A smooth curve can be approximated by a polygonal made up of segments of geodesics, and parallel transport along the curve is given a precise meaning as the result of parallel translation along these segments. Let $\kappa_g(s)$ be the geodesic curvature of the curve, in terms of the arc-length parameter, and $\alpha(s)$ the angle between the vector in parallel transport along the curve and the tangent vector to the curve; we have⁷

$$\frac{d\alpha(s)}{ds} = \kappa_g(s). \quad (1)$$

(c) If a vector at point A is parallel transported to B along two different curves from A to B , the end vectors at B are in general different.

(d) If a vector at A is transported along a small closed loop and returns to the starting point A , the end vector has rotated by a small angle θ relative to its original position. The angle θ is proportional to the area enclosed by the loop, and the coefficient of proportionality is the Gaussian curvature of the surface at A .

Familiarity with these properties is essential for the understanding of the corresponding results in three or more dimensions, and also for the case where the transformations associated with closed loops (holonomy transformations) are not Euclidean rotations but, e.g., Lorentz transformations, as in General Relativity.

The remainder of this paper shows how the SSC can be successfully employed to "see" properties (a)–(d) at work. Section II introduces the Chinese South-Seeking chariot and gives the fundamentals for explaining its striking behavior. Only elementary Euclidean geometry is required to understand this behavior on a flat plane. Section III discusses what happens when the SSC is allowed to explore a nonflat surface; the ideas are quite similar and the mathematics required to supply the proofs are given in Appendix B. Section IV contains some directions on the experimental exploration of geometry of surfaces with the help of the SSC. Finally, Appendix A explains why the



Fig. 1. A Chinese South-Seeking chariot.

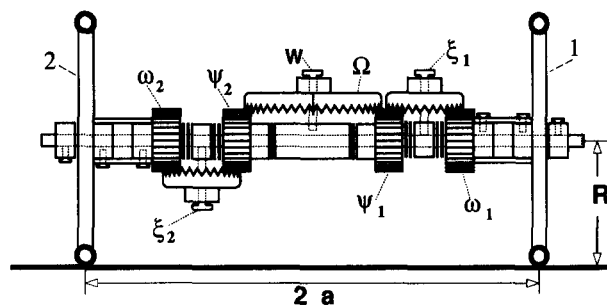


Fig. 2. Diagram of gearing of the South-Seeking chariot in Fig. 1. Only the two collars at both ends and the two axis ξ_1, ξ_2 are fixed to the main horizontal axis in order to hold the chariot together; all the other gears and the two assemblies wheel 1-gear ω_1 (resp. 2- ω_2) should be loose on their axes. The axis W should be in vertical position, not fixed to the main axis. Lateral play should be kept to a minimum, by means of thin spacing washers which also help in enforcing the condition $a=2R$.

particular design of SSC used here fulfills the basic requirements to work properly.

II. THE SOUTH-SEEKING CHARIOT

The Chinese South-Seeking Chariot is a mechanical device installed in a one-axis, two-wheeled chariot. It governs the position of an horizontal pointer in such a way that regardless of the motion of the chariot on a flat surface, at any moment the rate of rotation of the pointer relative to the chariot itself is exactly the opposite of the rate of rotation of the chariot relative to the ground. This arrangement leads to the property which justifies its name: regardless of the motion of the chariot on a flat surface, the pointer keeps always the same direction and when the chariot is dragged along a closed loop, on completion of the circuit the pointer returns to its original direction.

The SSC is a truly fascinating mechanical device and amazingly, it is indeed a very old mechanism. Its origin is Chinese, and there has been some controversy on the time of its invention. Some authors date it earlier,⁸ but according to J. Needham,⁹ it dates back to the third century and its inventor was likely Ma Chün. It is the oldest known self-regulating mechanism, and in this sense a precursor of all cybernetical machines. The mechanical arrangement has the differential gear as a basic building block, and must be considered a remarkable achievement. Some historians of technology have claimed that the chariot was actually used as a kind of nonmagnetic compass. This is, however, quite doubtful since correct performance relies (see Appendix A) on sensitive adjustments of the width between the wheels and also of a perfect contact of the wheels with the ground, without sliding or jumping.

A description with some schemes of a version of the mechanism can be found in Strandh.¹⁰ Figure 1 shows a very simple but efficient design¹¹ and Fig. 2 gives a diagram of the mechanism of this SSC, which is explained in Appendix A. Written descriptions of the mechanism tend to be tedious and inexpressive. Yet a working unit can be

easily built either by using only standard Meccano-like parts—as the one in Fig. 1—or from purposely made (or adapted) gears and wheels. It is worthwhile to build one and see it at work.

What is the reason for the intriguing behavior of the SSC? The answer rests both on the mechanism design and on plane Euclidean geometry. Let us first discuss the bearing on the design. By *path* of the SSC (Fig. 3) we will mean the path of the middle point of its axis. If $2a$ is the separation between the wheels, each of these will follow a “railway track” with two curves equidistant (“parallel”) to the path at (oriented) distances a and $-a$. The SSC “feels” its path only through the lengths traversed by its wheels. Let us denote $L(d;s)$ the length of the curve parallel to the path at distance d , and $\alpha(s)$ the angle turned by the pointer relatively to some reference direction fixed to the chariot, both as functions of the arc length s on the path. The design of the SSC must guarantee the relation

$$\alpha(s) = (1/2a) [L(a;s) - L(-a;s)]. \quad (2)$$

How this is accomplished in the SSC of Figs. 1 and 2 is explained in Appendix A. The rest of the story is plane Euclidean geometry: For any smooth curve Γ parametrized by its arc length s , let $\mathbf{v}(s)$ be a field of parallel vectors (i.e., its Cartesian components are constant) defined on the curve, $\kappa(s)$ the curvature of the curve, and $\alpha(s)$ the angle between $\mathbf{v}(s)$ and the tangent vector at the point s ; we have

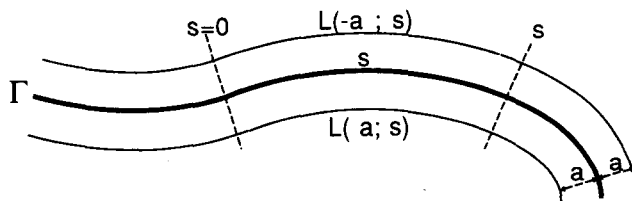


Fig. 3. The geometry of the path of the South-Seeking chariot. Γ is the middle path, and the tracks of the two wheels are two equidistant parallels, at distances a and $-a$. The difference of length of corresponding sections of the tracks is related to the curvature of Γ by means of Eqs. (B9) or (4).

$$\frac{d\alpha(s)}{ds} = \kappa(s). \quad (3)$$

Consider on the other hand two curves parallel to Γ at distances a and $-a$. The derivative of the difference of lengths between corresponding sections on both tracks, with respect to the path length, is approximately:

$$\frac{d}{ds}[L(a;s) - L(-a;s)] \approx 2a\kappa(s) \quad (4)$$

and the approximation is better and better for the separation a smaller and smaller. This equation is derived in Appendix B for curves on an arbitrary surface, but it can be easily understood in some particular examples for the flat case. On a straight section of the path, $L(a;s) = s$ independently of a , so both members in (4) vanish. For a circular section of the path, with radius r and constant curvature $\kappa = 1/r$, $L(a;s) = s + (sa/r)$, and the lhs of (4) equals $2a\kappa$. In both cases (4) is an equality. When the path has variable curvature, any small part of it can be approximated by an arc of its osculating circle. The straight and circular cases above suggest that the approximation in (4) is always a good one; this is indeed true and the exact version (B9) is

$$\left. \frac{d}{ds} \frac{dL(d;s)}{dd} \right|_{d=0} = \kappa(s). \quad (5)$$

All the pieces required to understand the behavior of the SSC are now at hand. When the path is straight ($\kappa = 0$), the two wheels traverse the same length for each segment of the path. The mechanism [Eq. (2)] ensures $\alpha(s) = 0$ so the pointer does not turn at all relative to the chariot; as the chariot itself does not turn relative to the ground, Eq. (3) holds. When the track is an arc of circle with constant curvature $\kappa \neq 0$, the wheels turn at different rates, and Eq. (2) gives $\alpha(s) = s/r$, so that Eq. (3) also holds. In both cases, the pointer remains in the same direction relative to the ground. For the general case of a path with variable curvature, substitution of (2) into (4) shows that

$$\frac{d\alpha(s)}{ds} \approx \kappa(s).$$

In the limit $a \rightarrow 0$, that equation goes into the equality (3), which characterizes (insofar direction changes are concerned) parallel transport on the plane. We conclude that in this limit the SSC performs parallel transport. All this should have been clear to the mind of its designer, of course with other words.

III. THE SOUTH SEEKING ON A SURFACE

But maybe the designer did not foresee what could happen if the SSC is now taken out of the flat surface of a table. Let us try to put it on a curved surface: at first sight the SSC is fooled and the property of the pointer returning back to the same direction after traversing a closed circuit no longer holds. Apparently the SSC fails from its original purpose of "keeping the same direction." However this apparent failing is actually a virtue in disguise, as in fact the SSC is actually "keeping the same direction" at a deeper level which reveals the curvature of the surface! The

specification of design of the SSC embodied in Eq. (2) refers to quantities which still makes sense for the new situation: the lengths of the tracks of the two wheels of the chariot, and the angle turned by the pointer relative to the chariot. The chariot will be ignorant about its new environment (the curved surface) and will blindly follow Eq. (2), using as input data the real lengths measured by its wheels. For curves on a surface, Eq. (4) becomes, for small a :

$$\frac{d}{ds}[L(a;s) - L(-a;s)] \approx 2a\kappa_g(s), \quad (6)$$

where $\kappa_g(s)$ is the geodesic curvature of the path at point s (see Appendix B). The basic equation of the chariot (2) implies that the function $\kappa_g(s)$ still satisfies

$$\frac{d\alpha(s)}{ds} \approx \kappa_g(s) \quad (7)$$

with an equality in the limit $a \rightarrow 0$. Whereas Eq. (3) determines the field of parallel directions to the curve with curvature $\kappa(s)$ in the Euclidean plane, Eq. (7) does the same for the field of geodesically parallel directions along the curve with geodesic curvature $\kappa_g(s)$ on the surface.⁷ This completes the proof that a unit vector on the pointer of the SSC undergoes (in the limit $a \rightarrow 0$) parallel transport for any arbitrary motion of the chariot on any surface.

IV. EXPERIMENTAL GEOMETRY

A. The South-Seeking chariot as a measuring instrument on a surface

The SSC can be used to experimentally check the features (a)–(d) of curvature and parallel transport listed in the Introduction, and it can even be used to measure both the geodesic curvature of any curve on a surface and the Gaussian curvature of the surface itself. In both cases it provides a local procedure for the measurement of these quantities. Let us first discuss the (geodesic) curvature of curves. If the SSC moves on a surface in such a way that the pointer remains stationary relative to the chariot itself, its path will be a geodesic [for then Eq. (7) implies $\kappa_g = 0$]. The same applies to the motion without sliding of a simple chariot whose two wheels are rigidly connected to the axis. This is so because of the geodesics property of being "frontal lines";¹² according to Eqs. (3) and (4) to prevent any differential rotation of the wheels while avoiding slippage implies that the pointer is stationary relative to the chariot.

If the path curve induces some rotation of the pointer relative to the chariot, the geodesic curvature of the curve is different from 0. A simple measure of the curvature can be given as the rate of change (respect to the path length) of the angle between the pointer and the chariot [Eq. (7)].

Consider now a smooth, closed, and simple circuit Γ in the surface, enclosing a domain D which has Γ as boundary. Parallel transport of a vector along Γ will return a vector which has rotated by some angle Δ . Equation (7) gives the rate of change of the angle between a vector in parallel transport along any curve and the tangent vector to the curve; when applied to Γ , whose tangent vectors at the beginning (say $s = s_0$) and at the end ($s = s_1$) coincide, it gives

$$\Delta = \int_{s_0}^{s_1} \kappa_g(s) ds. \quad (8)$$

The angle Δ is in general different from 2π and the deviation depends on the circuit as a whole. Experimentation with the chariot soon reveals this. If we put $\Delta = 2\pi - \theta$, then θ is the angle turned by the pointer after completion of the circuit, relative to the chariot itself. This angle is best calculated through the Gauss-Bonnet theorem¹³

$$\iint_D K(x^1, x^2) dS + \int_{s_0}^{s_1} \kappa_g(s) ds = 2\pi. \quad (9)$$

(If Γ is only piecewise smooth and has corners, the angles at the corners have to be considered too.) Equation (8) then gives:

$$\iint_D K(x^1, x^2) dS = \theta, \quad (10)$$

so the angle of the rotation induced by parallel transport along Γ is the integral of the Gaussian curvature over the domain D . If K is nearly constant on D (this will always be the case for small enough domains in smooth surfaces), a good approximation to the surface integral is K times the (oriented) area S of D . The point of interest here is that, as long as the SSC measures directly the angle θ , and checks the proportionality between θ and S for small loops, it also gives an indirect measure of the Gaussian curvature of the surface as the proportionality coefficient between θ and S .

B. Experimental exploration of the intrinsic geometry of surfaces with the SSC

The SSC can be used to explore "experimentally" the geometric properties of surfaces (of constant or variable curvature). From blocks of expanded polystyrene, measuring say $80 \times 80 \times 25$ cm, models of different surfaces can be cut (using a hot wire) very easily; with the help of a simple and suitable mechanical setting, specific surfaces (e.g., of constant curvature) could also be made very accurately. A sector of a sphere is the immediate choice for positive curvature: suitable values for the radius are 200, 100, and 50 cm with curvatures $K = 0.25$; 1, and 4×10^{-4} cm⁻². For negative curvature things are not so easy, but convenient parts of a pseudosphere (revolution surface of the tractrix) with curvatures of the same absolute value can also be made. Various surfaces of zero curvature could be cut as well, for instance parts of cylinders and of cones of varying angles at the vertex, including or excluding the vertex. These are of course the easiest to cut. It is worthwhile to have also some examples of surfaces with variable curvature.

In each case geodesics can be realized as paths for which the pointer of the SSC remains stationary relative to the chariot. Any such curve can be traced on the surface, allowing an experimental check of the extremal length property and of other local properties as the geodesic deviation between nearby geodesics, so relevant in general relativity.

Along any arbitrary curve, the pointer will not remain stationary relative to the chariot; the rate of change of the

angle of the pointer with the length of the middle path (the given curve) gives a measure of the geodesic curvature of the curve.

A triangle with area S made up three arcs of geodesics has an (positive, null, or negative) angular excess ϵ . For surfaces of constant curvature K , $\epsilon = KS$, so for a triangle of 2×10^3 cm² of area on a sphere of $r = 50$ cm of radius, the excess is of the order of $\pi/4$, and hence very noticeable. Follow this triangle with the SSC. When it comes back to the starting position, the pointer has rotated an angle equal to ϵ . Note the behavior of the pointer at the corners, where the SSC does not advance but only turns. Experimentation with the SSC easily shows: (1) that the angle turned by the pointer does depend only on the area of the enclosed triangle but not on its shape, (2) that the sense of the rotation is the same or opposite to the sense of traveling around the circuit according the curvature of the surface is positive or negative, and (3) that the angle is proportional to the area of the enclosed triangle.

Similar experiments on surfaces of constant curvature for general simple closed loops show that the angle of rotation depends only on the area enclosed by the path, but is independent of its shape. Thus the proportionality stated in Sec. I(d) can be "experimentally" checked. If the surface does not have constant curvature, the relevant quantity [Eq. (10)] is the surface integral of K . Particularly dramatic is the case of a portion of a cone including the vertex: When completing a simple closed circuit, the pointer either has turned by some fixed angle or it has returned to its initial position, according to whether the path encloses the vertex or not. If β is the cone semiangle at the vertex, there is a "Dirac's delta" curvature concentrated in the vertex giving a finite value $2\pi(1 - \sin\beta)$ to the integral in (10).

C. The South-Seeking chariot, LOGO, and computers

Another interesting idea is to couple the SSC to a computer. Adequate analog or digital devices can inform the computer of the rotation angles of the wheels. These contain all the information the SSC needs to work, and also measure the length of the middle path (the half-sum of the lengths traversed by the wheels). From integration with respect to the length the computer can derive immediate information on the angle turned by the pointer of the chariot, the geodesic curvature of the path, etc. (Notice the resemblance, *mutatis mutandis*, with the measurement of velocity changes by time integrator of the reading of an accelerometer¹⁴ which yields the "good" rapidity concept in relativity.) There are some obvious similarities of this project with the LOGO turtle,¹⁵ which was designed as a way to explore experimentally Euclidean plane geometry. While the LOGO turtle can also be used to explore a curved "screen," this requires previous knowledge of the relevant facts of differential geometry, to be built-in in the procedures which drive the turtle. On the other hand, the SSC shows directly, not through any simulation, these relevant facts. In this sense the LOGO turtle and the SSC are complementary tools in an experimental exploration of the geometry of curved surfaces.

V. CONCLUSION

Apart from its interest for the history of technology, the SSC provides a useful pedagogical tool: it is inexpensive, easy to construct, and it makes a fascinating demonstration

of the properties of curvature and parallel transport, stimulating discussion of further interesting questions. The intuitive understanding provided through experimentation with the SSC will help to make the ideas of curvature and parallel transport more easily understood and remembered.

ACKNOWLEDGMENTS

This work has been supported partially by CICYT (Spain) under reference AEN90-0027.

APPENDIX A: THE DESIGN OF THE CHARIOT

The only requirement to be met when designing a SSC is that it fulfills the basic equation (2). There is considerable freedom in the choice of the mechanism itself.¹⁶ The standard version has two equal wheels and utilizes coupled differentials in some way. Each wheel of radius R transforms the length along its track into a rotation angle $\beta(a;s) = (1/R)L(a;s)$. Therefore Eq. (2) gives

$$\alpha(s) = (R/2a)[\beta(a;s) - \beta(-a;s)]. \quad (A1)$$

The sum of the rotations of the two wheels must be absorbed into the mechanism and the difference (stepped by the factor $R/2a$) must be transferred to the pointer. Notice that this is the opposite of the requirement for a car differential. As the stepping factor is fixed by the details of the mechanism design, the chariot will not perform correctly unless $R/2a$ coincides very accurately with this stepping value. Any slight variation in the separation between the wheels, as well as an occasional loss of contact of a wheel with the ground will produce an unwanted drift of the pointer.

In the chariot of Figs. 1 and 2, $a = 2R \approx 6$ cm, so $R/2a = \frac{1}{4}$. The gear train from the wheels to the pointer must transfer to the pointer one fourth of the difference of the rotation of the wheels. This is attained by using three coupled differentials, one for each wheel and a central one. Let us denote by the same symbols the gears and their angular speeds around the common axis in Fig. 2. Note that the cages of the differentials revolves around this axis with angular speeds ξ_i , while the angular speeds of the idle gears whose axis make up the cage is irrelevant. Let Ω denote the angular speed of the wheel which carries the pointer itself (the idler on the central differential), relatively to the frame of the chariot. The assembly of the wheel i and the gear ω_i revolves as a whole loosely on the axle, and therefore ω_i is equal to the angular speed of the wheel i ($i=1,2$). Two differentials, with ω_i, ψ_i as external axles, and ξ_i as cage carry the motion of the wheels into a third (central) differential. The two cages ξ_i are fixed to the axle; as the axle itself is not coupled to any other gear, this has only the effect of enforcing the equality $\xi_1 = \xi_2$. The two gears ψ_i are also loose on the axis, and both serve as external entries for the third differential, whose cage is fixed in a vertical position (see the photograph Fig. 1), so that $W=0$, whereas Ω is loose on its axle. The gears ω_i, ψ_i, ξ_i have all the same number of teeth (25 in our model), but Ω has twice this number. The equations for the angular speeds are

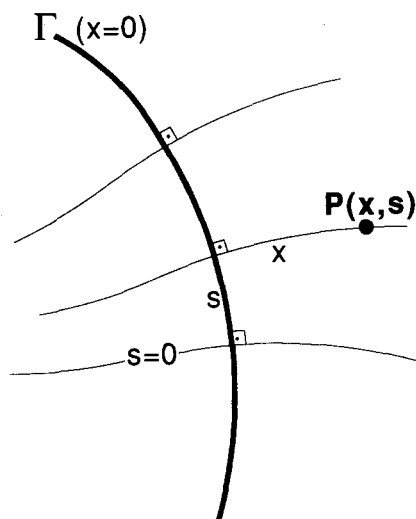


Fig. 4. Geodesically parallel or Fermi coordinates based on the curve Γ . Coordinates (x,s) of a point P are determined as lengths through the geodesic by P orthogonal to Γ , and along Γ itself.

Wheel differential i ($i=1,2$)	$\omega_i + \psi_i = 2\xi_i$
Linkage between the cages 1 and 2	$\xi_1 = \xi_2$
Central differential	$\psi_1 + \psi_2 = 2W$,
Cage of central differential fixed	$W = 0$,
Pointer (twice as many teeth as $\psi_{1,2}$)	$2\Omega = \psi_1 = -\psi_2$.

(A2)

A quite simple algebraic calculation shows that $4\Omega = \omega_2 - \omega_1$, as required.

APPENDIX B: PROOF OF EQS. (4) AND (6)

In this appendix we provide proofs of Eqs. (4) and (6) using elementary differential geometry. These equations link the difference of lengths of two parallel tracks on any surface to the geodesic curvature of the (middle) path.

An adequate choice of coordinates is helpful. Here, the best ones are the so-called geodesically parallel or Fermi coordinates, with base in the middle path Γ .¹⁷ The coordinates (x,s) are always well defined on some strip of surface along the middle path. For any point P in this strip, draw the geodesic orthogonal to Γ through P , and let Q be the point of intersection of this geodesic with Γ . The coordinate x is the length of the geodesic arc between P and Q , and s is the length of the arc of the path Γ between Q and some reference point on Γ (see Fig. 4).

The metric tensor is given in such a coordinate system as

$$dl^2 = dx^2 + G(x,s)ds^2 \quad (B1)$$

and the function $G(x,s)$ satisfies:

$$G(0,s) = 1. \quad (B2)$$

The Christoffel symbols can be easily calculated. We shall only need

$$\Gamma_{ss}^x(x,s) = -\frac{1}{2} \frac{\partial G(x,s)}{\partial x}. \quad (B3)$$

The Euclidean plane corresponds to $G(x,s)=1$, and (x,s) are then ordinary Cartesian coordinates, with vanishing Christoffel symbols.

Let us take the two tracks at distances a and $-a$ from the base curve Γ . For small enough a we can write

$$L(a;s) - L(-a;s) \approx 2a \left. \frac{\partial L(a;s)}{\partial a} \right|_{a=0} \quad (\text{B4})$$

The parametric description of the tracks is $s \rightarrow (a,s)$, and their length $L(a;s)$ between parameters 0 and s is given by

$$L(a;s) = \int_0^s [G(a,s)]^{1/2} ds. \quad (\text{B5})$$

Compute now the derivative in (B4) using (B5) and (B2). This gives

$$\left. \frac{\partial L(a;s)}{\partial a} \right|_{a=0} = \frac{1}{2} \int_0^s \left. \frac{\partial G(a;s)}{\partial a} \right|_{a=0} ds. \quad (\text{B6})$$

The next task is to relate this value with the geodesic curvature of the path. It is a standard result¹⁸ that for a general metric $g_{ij}(x^1, x^2)$, the geodesic curvature of the coordinate curves $x^1 = \text{const}$ (say $x^1 = a$) is a function of x^2 given by

$$\kappa_g(a, x^2) = -\Gamma^1_{22}(a, x^2) (g^{1/2}/g_{22}^{3/2}), \quad (\text{B7})$$

As the path of the chariot is the base curve Γ , (B7) and (B2) give

$$\kappa_g(s) = -\Gamma^x_{ss}(0,s) = \frac{1}{2} \left. \frac{\partial G(x,s)}{\partial x} \right|_{x=0} \quad (\text{B8})$$

and hence by substitution into (B4) and (B6):

$$L(a;s) - L(-a;s) \approx 2a \left. \frac{\partial L(a;s)}{\partial a} \right|_{a=0} = 2a \int_0^s \kappa_g(s) ds. \quad (\text{B9})$$

By taking the derivative with respect to s one obtains (6) for a general surface or (4) for the flat Euclidean case.

¹R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison Wesley, Reading, MA, 1964), Vol. II, Chap. 42.
²M. V. Berry, "The geometric phase," *Sci. Am.* **259** (12), 46-55 (1988); J. B. Hart, R. E. Miller, and R. L. Mills, "A simple geometric model for visualizing the motion of a Foucault pendulum," *Am. J. Phys.* **55**, 67-70 (1987).
³M. Kugler, "Motion in noninertial systems: Theory and demonstrations," *Am. J. Phys.* **57**, 247-251 (1989).
⁴M. V. Berry, "Quantal phase factors accompanying adiabatic changes," *Proc. R. Soc. London Ser. A* **392**, 45-57 (1984).
⁵A. Shapere and F. Wilzeck, Eds., *Geometric Phases in Physics*, Advanced Series in Mathematical Physics, Vol. 5 (World Scientific, Singapore, 1989).
⁶Yu. I. Manin, *Mathematics and Physics*, Progress in Physics, Vol. 3 (Birkhäuser, Boston, 1981), p. 21.
⁷See, e.g., M. P. Do Carmo, *Differential Geometry of Curves and Surfaces* (Prentice Hall, Englewood Cliffs, NJ, 1976), Sec. 4-4, p. 249.
⁸For instance, the "Machines" article of *Encyclopedia Britannica*, 1982 ed. dates as the earliest known gears those found in a Chinese South-Pointing chariot from the 27th century B.C.
⁹J. Needham, *The Grand Titration. Science and Society in the East and the West* (George Allen & Unwin, London, 1969), Chap. 2, II, Footnote 62.
¹⁰S. Strandh, *Machines: an Illustrated History* (Nordbok, Gothenburg, Sweden, 1982), Chap. II, p. 44.
¹¹This particular SSC has been designed by John Nuttall and published by N. C. Ta'Bois, "Chinese South Seeking Chariots," *Meccano Mag.* **64**, 128-130 (1979).
¹²D. Hilbert and S. Cohn-Vossen, *Geometry and the Imagination* (Chelsea, New York, 1952), p. 220.
¹³See, e.g., E. Kreyszig, *Introduction to Differential Geometry and Riemannian Geometry* (Univ. of Toronto, Canada, 1968), Chap. X, Sec. 69, p. 209-210.
¹⁴J. M. Levy-Leblond, "Speed(s)," *Am. J. Phys.* **48**, 345-347 (1980).
¹⁵See, e.g., H. Abelson, and A. di Sessa, *Turtle Geometry* (Massachusetts Institute of Technology, Cambridge, MA, 1980).
¹⁶Other articles on South Seeking chariots giving alternative designs are T. Morris, "South-Seeking Chariot," *Meccano Mag.* **62**, 30-31 (1977) and A. Partridge, "Chinese Chariot: Economy drive," *Meccano Mag.* **63**, 74-75 (1978); there one can learn that it is even possible to build a gearless SSC. See also M. Denny, "Chinese South-Facing Chariot," *Constructor Q.* **3**, 46-48 (1989), and A. Partridge, "The mandarin rides again," *Constructor Q.* **5**, 44-45 (1989).
¹⁷Reference 13, Chap. VII, Sec. 45, p. 145-146.
¹⁸Reference 13, Chap. VII, Sec. 42, p. 139.

JOHN BELL—THE AIM OF NATURAL PHILOSOPHY

In the beginning natural philosophers tried to understand the world around them. Trying to do that they hit upon the great idea of contriving artificially simple situations in which the number of factors involved is reduced to a minimum. Divide and conquer. Experimental science was born. But experiment is a tool. The aim remains: to understand the world. To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise. A serious formulation will not exclude the big world outside the laboratory.

John S. Bell, "Against 'Measurement,'" CERN-TH-5611/89 (December 1989).